The 4D Einstein-Gauss-Bonnet theory of gravity

and PBH remnants as a dark matter candidate

Pedro G. S. Fernandes
In collaboration with Pedro Carrilho, Timothy Clifton and David Mulryne

London Cosmology Discussion Meeting (LCDM)

Outline

1. Lovelock's Theorem and The Gauss-Bonnet Term
2. Gauss-Bonnet dynamics in 4D - a dimensional regularization procedure
3. Black hole solutions, a uniqueness theorem and PBH remnants as dark matter
4. Conclusions

Lovelock's Theorem

Assuming a D dimensional spacetime, diffeomorphism invariance, metricity and second-order equations of motion, Lovelock's theorem implies that the most general theory of gravity is described by [Lovelock, 1971]

$$S_D = \int d^D x \sqrt{-g} \sum_j \alpha_j \mathcal{R}^j, \qquad \mathcal{R}^j \equiv \frac{1}{2^j} \delta^{\mu_1 \nu_1 \ldots \mu_j \nu_j}_{\alpha_1 \beta_1 \ldots \alpha_j \beta_j} \prod_{i=1}^{j} R^{\alpha_i \beta_i}{}_{\mu_i \nu_i}$$

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The above action changes depending on the number of spacetime dimensions

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$$S_2 = \int d^2 x \sqrt{-g}\alpha_0, \quad S_3 = \int d^3 x \sqrt{-g}\left(\alpha_0 + \alpha_1 R\right), \quad S_4 = \int d^4 x \sqrt{-g}\left(\alpha_0 + \alpha_1 R\right),$$

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$$S_5 = \int d^5 x \sqrt{-g} \left(\alpha_0 + \alpha_1 R + \alpha_2 \mathcal{G} \right), \qquad \mathcal{G} = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta},$$

$\mathcal{G}$ is called the "Gauss-Bonnet term".

Equations of Motion and the Gauss-Bonnet Term in 4D

Consider now the Einstein-Gauss-Bonnet theory in D-dimensions

$$S_D = \frac{1}{16\pi G} \int d^D x \sqrt{-g} \left(-2\Lambda + R + \alpha \mathcal{G}\right) + S_M,$$

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Extremization of the action gives the equations of motion

$$\Lambda g_{\mu\nu} + G_{\mu\nu} + \alpha H_{\mu\nu} = 8\pi G T_{\mu\nu}$$

where

$$H_{\mu\nu} = 2R_{\mu}{}^{\alpha\rho\sigma} R_{\nu\alpha\rho\sigma} - 4R^{\rho\sigma} R_{\mu\rho\nu\sigma} - 4R_{\mu}{}^{\rho} R_{\nu\rho} + 2R\, R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{G}.$$

Equations of Motion and the Gauss-Bonnet Term in 4D

In four-dimensions $H_{\mu\nu}$ **vanishes** identically for all metrics. This is a consequence of Chern's theorem: in 4D, the Gauss-Bonnet invariant is the Euler density – its integral is a topological invariant χ called the *Euler characteristic* [Chern, 1945]

$$\chi \propto \int d^4x \sqrt{-g}\mathcal{G}.$$

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Take the trace of the field equations:

$$g^{\mu\nu}\left(\Lambda g_{\mu\nu} + G_{\mu\nu} + \alpha H_{\mu\nu}\right) = 8\pi G g^{\mu\nu} T_{\mu\nu}$$

$$-D\Lambda + \frac{(D-2)}{2}R + \alpha\frac{(D-4)}{2}\mathcal{G} = -8\pi G\, T$$

Einstein Gauss-Bonnet Gravity - Static Black holes

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{D-2},$$

$$f(r) = 1 + \frac{r^2}{2\alpha(D-3)(D-4)}\left(1 - \sqrt{1 + \frac{8GM\alpha}{r^{D-1}}(D-3)(D-4)}\right),$$

Horndeski gravity and the Gauss-Bonnet term

Horndeski gravity is analogous to Lovelock gravity, but addresses the most general scalar-tensor theory with second-order equations of motion [Horndeski, 1974]

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right] + G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi_{\mu\nu}\phi^{\nu\lambda}\phi^{\mu}_{\lambda}\right].$$

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Couplings between the scalar field and curvature scalars are only allowed by Horndeski's theorem if the curvature scalar is either R or $\mathcal{G}$ [1901.07183].

String Theory and the Gauss-Bonnet term

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- M-theory compactified on a Calabi-Yau threefold down to $D = 5$ takes the form

$$S_{eff} = \int d^5x\sqrt{-g}\left(R + \frac{1}{16}c_2^{(I)}V_I\,\mathcal{G}\right).$$

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- The 1-loop 4D effective action for heterotic string theory in the Einstein frame is of the form

$$S_{eff} = \int d^4x \sqrt{-g} \left(R - \frac{1}{2} (\nabla \phi)^2 + \alpha' e^{\phi} \mathcal{G} \right).$$

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5. Higher-order curvature terms play a role in efforts to quantize/renormalize gravity
6. We know that GR is only a low-energy EFT and it should be modified at some energy scale

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Dimensional Regularization of the Gauss-Bonnet term (2004.08362)

PHYSICAL REVIEW D **102,** 024025 (2020)

Derivation of regularized field equations for the Einstein-Gauss-Bonnet theory in four dimensions

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(Received 21 April 2020; revised 15 June 2020; accepted 24 June 2020; published 6 July 2020)

We propose a regularization procedure for the novel Einstein-Gauss-Bonnet theory of gravity, which produces a set of field equations that can be written in closed form in four dimensions. Our method consists of introducing a counterterm into the action, and does not rely on the embedding or compactification of any higher-dimensional spaces. This counterterm is sufficient to cancel the divergence in the action that would otherwise occur, and exactly reproduces the trace of the field equations of the original formulation of the theory. All other field equations display an extra scalar gravitational degree of freedom in the gravitational sector, in keeping with the requirements of Lovelock's theorem. We discuss issues concerning the equivalence between our new regularized theory and the original.

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Inspired by:

- Glavan and Lin's singular regularization of the Gauss-Bonnet term in 4D [1905.03601, PRL]

$$S = \lim_{D\to 4} \int d^D x \sqrt{-g} \left(R + \frac{\alpha}{D-4} \mathcal{G} \right)$$

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- Mann and Ross's dimensional regularization of the Ricci scalar in $2D$ [gr-qc/9208004, CQG]

Toy Model: Dimensional regularization of the Ricci scalar in 2D

In 2D the Ricci scalar has a topological character similar to the Gauss-Bonnet term in 4D: $\chi \sim \int d^2x\sqrt{-g}R$. We will apply a regularization procedure as follows.

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2. Perform a singular rescaling of the coupling à lá Glavan and Lin

$$S = \frac{\kappa}{D-2} \int d^D x \sqrt{-\tilde{g}}\tilde{R}.$$

3. Although the trace equation may be finite in the 2D limit, the theory is not well-defined. We will add a counterterm to remove divergences.

$$S = \frac{\kappa}{D-2}\int d^D x\sqrt{-\tilde{g}}\tilde{R} - \frac{\kappa}{D-2}\int d^2 x\sqrt{-\tilde{g}}\tilde{R}.$$

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4. Take the 2D limit

$$S = \kappa \lim_{D \to 2} \frac{\int d^D x \sqrt{-\tilde{g}} \tilde{R} - \int d^2 x \sqrt{-\tilde{g}} \tilde{R}}{D-2} \to \frac{0}{0}$$

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5. Evaluate the limit by noting that it is a derivative

$$S = \kappa \lim_{D\to 2} \frac{\int d^D x\sqrt{-\tilde{g}}\tilde{R} - \int d^2 x\sqrt{-\tilde{g}}\tilde{R}}{D-2} = \kappa \frac{d}{dD}\left(\int d^D x\sqrt{-\tilde{g}}\tilde{R}\right)\Big|_{D=2}$$

Toy Model: Dimensional regularization of the Ricci scalar in 2D

6. Taking a dimensional derivative sounds kind of odd. Let us, however, introduce a conformal metric $g_{\mu\nu} = e^{-2\phi}\tilde{g}_{\mu\nu}$, for which

$$\sqrt{-\tilde{g}}\tilde{R} = \sqrt{-g}e^{(D-2)\phi}\left[R - 2(D-1)\Box\phi - (D-1)(D-2)\left(\nabla\phi\right)^2\right]$$

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Taking the dimensional derivative and evaluating the result in 2D gives

$$S = \kappa \frac{d}{dD}\left(\int d^D x \sqrt{-\tilde{g}}\tilde{R}\right)\Big|_{D=2} = \kappa \int d^2 x \sqrt{-g}\left(\phi R + \left(\nabla\phi\right)^2\right).$$

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7. The theory is well-defined in 2D, its field equation decouples from the scalar and has a GR-like behavior

$$R = \frac{2}{\kappa}T.$$

Dimensional Regularization of the Gauss-Bonnet term (2004.08362)

Consider the previous regularization procedure applied to the Gauss-Bonnet term in the 4D limit. After following the same steps, one can arrive at the well-defined action

$$S = \int d^4x\sqrt{-g}\left[R - \alpha\left(\phi\mathcal{G} - 4G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - 4\Box\phi(\nabla\phi)^2 - 2(\nabla\phi)^4\right)\right] + S_M,$$

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The theory belongs to the shift-symmetric ($\phi \to \phi + c$) Horndeski class of theories

$$G_2 = 8\alpha X^2, \qquad G_3 = 8\alpha X, \qquad G_4 = 1 + 4\alpha X, \qquad G_5 = 4\alpha \log X.$$

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$$G_2 = 8\alpha X^2, \qquad G_3 = 8\alpha X, \qquad G_4 = 1 + 4\alpha X, \qquad G_5 = 4\alpha \log X.$$

Remarkably, a linear combination of the trace and scalar field equations leads to

$$R + \frac{\alpha}{2}\mathcal{G} \propto -T.$$

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A uniqueness theorem and PBH remnants as dark matter (2107.00046)

PHYSICAL REVIEW D **104,** 044029 (2021)

Black holes in the scalar-tensor formulation of 4D Einstein-Gauss-Bonnet gravity: Uniqueness of solutions, and a new candidate for dark matter

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(Received 5 July 2021; accepted 16 July 2021; published 11 August 2021)

In this work we study static black holes in the regularized 4D Einstein-Gauss-Bonnet theory of gravity; a shift-symmetric scalar-tensor theory that belongs to the Horndeski class. This theory features a simple black hole solution that can be written in closed form, and which we show is the unique static, spherically symmetric and asymptotically flat black hole vacuum solution of the theory. We further show that no asymptotically flat, time-dependent, spherically symmetric perturbations to this geometry are allowed, which suggests that it may be the only spherically symmetric vacuum solution that this theory admits (a result analogous to Birkhoff's theorem). Finally, we consider the thermodynamic properties of these black holes, and find that their final state after evaporation is a remnant with a size determined by the coupling constant of the theory. We speculate that remnants of this kind from primordial black holes could act as dark matter, and we constrain the parameter space for their formation mass, as well as the coupling constant of the theory.

DOI: 10.1103/PhysRevD.104.044029

The static black hole solution

Consider the regularized theory on a general static and spherically symmetric background of the form

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

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The field equations allow the following black hole solution

$$A(r) = B(r)^{-1} = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8M\alpha}{r^3}}\right), \quad \phi = \int \frac{1-\sqrt{A(r)}}{r\sqrt{A(r)}}dr$$

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The Gauss-Bonnet corrections weaken the central singularity – in GR, the Kretschmann scalar diverges as $\sim r^{-6}$, whereas in 4D EGB it diverges as $\sim r^{-3}$. Moreover, the black hole entropy presents a logarithmic correction.

A uniqueness theorem (2107.00046)

- A remarkable results holds for this theory: assuming staticity and asymptotically flatness, it can be shown that this is the **unique** spherically symmetric vaccum black hole solution of the theory.

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- Let us now address time-dependence by considering time-dependent spherically symmetric perturbations about the black hole solution

$$A(t,r) = A_0(r) + \sum_{n=1}^{\infty} \epsilon^n A_n(t,r), \quad B(t,r) = B_0(r) + \sum_{n=1}^{\infty} \epsilon^n B_n(t,r),$$

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- In GR such perturbations are zero, by virtue of Birkhoff's theorem.

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- Remarkably, in our theory, assuming only asymptotic flatness, in can be shown that all perturbations vanish.
- This suggests that the above black hole solution is the **unique** asymptotically flat, spherically symmetric vacuum black hole spacetime of the theory, and that it is perturbatively **stable**.

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$$r_{\pm} = M \pm \sqrt{M^2 - \alpha},$$

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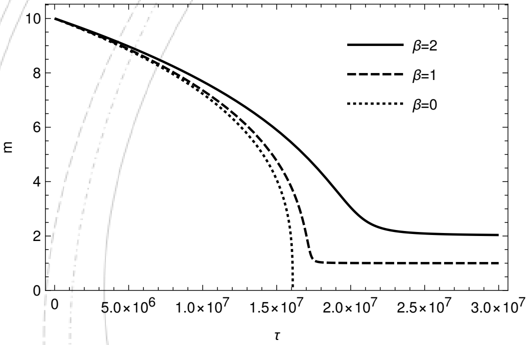
and a Hawking temperature given by

$$T = \frac{r_+^2 - \alpha}{4\pi r_+ \left(r_+^2 + 2\alpha\right)},$$

where we note that $T \to 0$ as $r_+ \to \sqrt{\alpha}$.

PBH remnants as dark matter (2107.00046)



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- Evaporation should lead to stable non-thermal remnants that interact only gravitationally.

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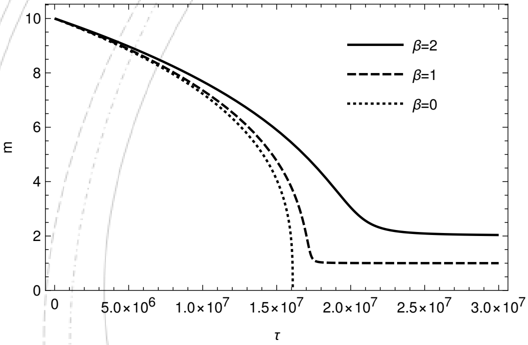

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- Evaporation should lead to stable non-thermal remnants that interact only gravitationally.
- Dark matter candidate?

PBH remnants as dark matter (2107.00046)

- Let us assume a scenario where a population of PBHs can form when large perturbations re-enter the horizon during the period of radiation domination after inflation ends.

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- We want estimate the allowed parameter range of the PBH mass at formation M_{PBH} and α such that **all dark matter** is composed of PBH remnants.

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For this scenario to be consistent we impose:

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- $z_{ev} > z_{eq} \approx 3400$ (to avoid relic production occurring after matter-radiation equality)

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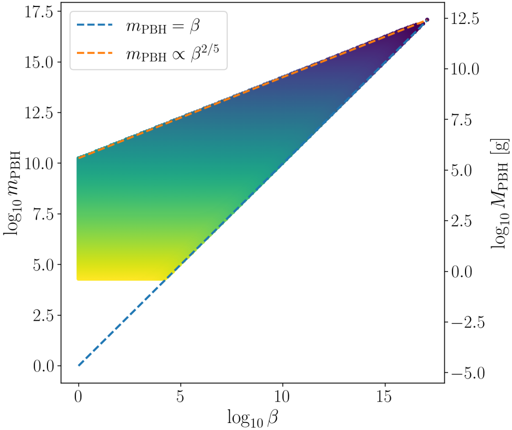

- PBH mass at formation can range from $M_{PBH} \approx 0.4$g to $M_{PBH} \approx 10^5$g when $\sqrt{\alpha} = l_{pl}$, and can reach $M_{PBH} \approx 10^{12}$g when $\sqrt{\alpha} \approx 10^{-18}$m, which is the maximum value of this coupling for which this scenario is valid.

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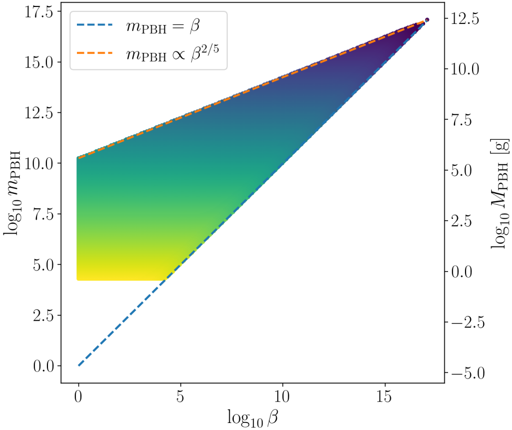

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- Current observations impose only a mild constraint $\sqrt{\alpha} \lesssim 10^4$ m $= 10$km [2006.15017].

Outline

1. Lovelock's Theorem and The Gauss-Bonnet Term
2. Gauss-Bonnet dynamics in 4D - a dimensional regularization procedure
3. Black hole solutions, a uniqueness theorem and PBH remnants as dark matter
4. Conclusions

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- Described and discussed a regularization procedure that enables non-trivial Gauss-Bonnet dynamics in 4D, similar to those of the higher-dimensional EGB theory.
- Showed that black hole solution in 4D Gauss-Bonnet gravity obeys a uniqueness theorem. Thus, if this theory is realized in Nature, static black holes must be described by this solution.

Conclusions

- Briefly discussed the thermodynamics of the black hole, showing that the Hawking temperature vanishes as $r_+ \to \sqrt{\alpha}$, leaving stable non-thermal remnants of size $\sqrt{\alpha}$.

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Conclusions

- Briefly discussed the thermodynamics of the black hole, showing that the Hawking temperature vanishes as $r_+ \to \sqrt{\alpha}$, leaving stable non-thermal remnants of size $\sqrt{\alpha}$.
- Speculated on the possibility that primordial evaporation remnants constitute **all** dark matter
  - Valid scenario if $\sqrt{\alpha} \lesssim 10^{-18}$m.
  - Remnants with a mass larger than the Planck mass (which follow when $\sqrt{\alpha} > l_{pl}$) allow for the formation of PBHs at lower energy scales than in the standard scenario of Planck mass remnants.

Thank you for your attention!