Cosmological implications of EW vacuum instability: constraints on the Higgs curvature coupling from inflation

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Introduction: Motivation

Experimental values of SM particle masses $m_h; m_t$ indicate that:

- SM may be valid up to $_{\mathrm{QG}}$; early Universe consistent minimal model.

$$V_{\mathrm{H}}(h; \;\;) = \frac{(\;\;)}{4} h^4$$

Markkanen *et al*, "Cosmological Aspects of Higgs Vacuum Metastability", 2018.

Motivation

Experimental values of SM particle masses $m_h; m_t$ indicate that:

- currently in metastable EW vacuum ! constrain fundamental physics.

$$V_{\mathrm{H}}(h; \ ; R) = \frac{(\ )}{2} R h^2 + \frac{(\ )}{4} h^4$$

Markkanen *et al*, "Cosmological Aspects of Higgs Vacuum Metastability", 2018.

Higgs vacuum decay

Decay expands at c with singularity within → true vacuum bubbles:

$$d\langle N \rangle = \; dV \;\Rightarrow\; \langle N \rangle = \int_{\text{past}} d^4x \sqrt{-g}\,(x)$$

Universe still in metastable vacuum no bubbles in past light-cone

$$P(N = 0) \propto e^{-\langle N \rangle} \sim O(1) \;\Rightarrow\; \langle N \rangle \lesssim 1$$

Low decay rate today, but higher rates in the early Universe.

$$\langle N \rangle = \frac{4}{3} \int_0^{N_{\text{start}}} dN \left(\frac{a_{\text{inf}}(\;_0 \;\; (N))}{e^N} \right)^3 \frac{(N)}{H(N)} \quad 1$$

Decay rates from instantons

Classical solutions to the tunneling process from false to true vacuu

High H 's during in ation, CdL! HM instanton with action di erence

$$B_{HM}(R) \qquad \frac{384\ ^{2}\ V_H}{R^2}$$

where $V_H = V_H(h_{bar}) \quad V_H(h_{fv})$: barrier height! decay rate

$$_{HM}(R) \qquad \frac{R}{12}^{2} e^{\ B_{HM}(R)}$$

Coleman (1977), Coleman and De Lucia (1980), Hawking and Moss (1987).

Minkowski terms to 3-loops, curvature corrections in dS at 1-loop:

$$V_H(h;\ ;R\ ) = \frac{(\ \ )}{2} R h^2 + \frac{(\ \ )}{4} h^4 + \frac{(\ \ )}{144} R^2 + \quad V_{loops}(h;\ ;R\ )\ ;$$

where the loop contribution can be parametrized as

$$V_{loops} = \frac{1}{64\ ^2} \sum_{i=1}^{31} \quad n_i M_i^4 \ \log \frac{|M_i^2|}{^2} \quad d_i \ + \frac{n_i^0 R^2}{144} \log \frac{|M_i^2|}{^2}$$

RGI: choose $=\ \ (h;R)$ such that $V_{loops}(h;\ \ ;R) = 0\ \ !$

$$V_H^{RGI}(h;R) = \frac{(\ \ \ (h;R))}{2} R h^2 + \frac{(\ \ \ (h;R))}{4} h^4 + \frac{(\ \ \ (h;R))}{144} R^2$$

Markkanen et al, \The 1-loop e ective potential for the Standard Model in curved spacetime", 2018.

Overview of computation

1. Calculate V_H and plug it in .
2. Choose in ationary model $V(\ )$ and calculate cosmological quantities
3. Complete calculation of $\langle N \rangle$ imposing the condition $\langle N \rangle$ 1.

$$\langle N \rangle = \frac{4}{3} \int_0^{N_{start}} dN \quad \frac{a_{inf}(\ _0 \quad (N))}{e^N} \quad ^3 \frac{(\ N)}{H(N)} \quad 1$$

4. Result: constraints on $_{\langle N \rangle = 1}$.

In ationary framework beyond slow-roll

General single- eld( ) in ationary model: space-time geometry determined by the Friedmann eq. and the eld's EoM in FRW:

$$H^2 = \frac{1}{3M_P^2}\left(\frac{1}{2}\dot{\ }^2 + V(\ )\right)$$

$$\ddot{\ } = -3H\dot{\ } - V'(\ )$$

In terms of e-foldings N and the in aton eld without SR:

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{V(\ )}{3M_P^2}\left[1 - \frac{1}{6M_P^2}\left(\frac{d\ }{dN}\right)^2\right]^{-1}$$

$$R = 6\left(\frac{\dot a^2}{a^2} + \frac{\ddot a}{a}\right) = 12H^2\left[1 - \frac{1}{4M_P^2}\left(\frac{d\ }{dN}\right)^2\right]$$

Numerical solution

System of coupled di erential equations:

$$\frac{d^2}{dN^2} = \frac{V(\ )^2}{M_P^2 H^2} \quad \frac{d}{dN} \quad M_P^2 \frac{V^0(\ )}{V(\ )}$$

$$\frac{d\tilde{}}{dN} = \quad \tilde{}(N) \quad \frac{1}{a_{inf} H(N)}$$

$$\frac{dhNi}{dN} = \quad (N) = \frac{4}{3} \quad a_{inf} \quad \frac{3{:}21e^{\ N}}{a_0 H_0} \quad \tilde{}(N)^{\ 3} \quad \frac{(\ N)}{H(N)}$$

where $\tilde{} = e^{\ N}$ and : conformal time and we set the end of in ation at

$$\left.\frac{\ddot{a}}{a}\right|_{=\ inf} = H^2 \left[1 \quad \frac{1}{2M_P^2} \left(\frac{d}{dN}\right)^2 \right]_{=\ inf} = 0$$

In ationary Models

Quadratic in ation, where $m = 1{:}4 \quad 10^{13}$ GeV, with

$$V(\ ) = \frac{1}{2}m^2\ ^2$$

Quartic in ation, where $\ = 1{:}4 \quad 10\ ^{13}$, with

$$V(\ ) = \frac{1}{4}\ \ ^4$$

Starobinsky in ation (Starobinsky-like power-law model), where $\ = 1{:}1 \quad 10\ ^{5}$, with

$$V(\ ) = \frac{3}{4}\ ^2 M_P^4 \quad 1 \quad e^{\ \sqrt{\frac{2}{3}}\frac{}{M_P}}\ \ ^2$$

Quadratic and quartic models are simple but not realistic; Starobinsky in ation complies with data and can link di erent in ationary models.

Overview of computation

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3. Complete calculation of $\langle N \rangle$ imposing the condition $\langle N \rangle \quad 1.$

$$\langle N \rangle = \frac{4}{3}\int_{0}^{N_{start}} dN \quad \left(\frac{a_{inf}(\ _0 \quad (N))}{e^N}\right)^3 \frac{(\ N)}{H(N)} \quad 1$$

4. Result: constraints on $\quad_{\langle N \rangle = 1}$.

Results: Bounds on

Results: Bubble nucleation time

If bubbles form at $N < 1$! bounds maybe unreliable due to B_{HM}^{dS}.

If bubbles form at N 60 ! bounds would depend on early times.

Results: Signi cance of the total duration of in ation

In ation can last for many orders of magnitude longer than 60 e-folds.

We study early time behavior by splitting the $\langle N \rangle$-integral

$$\langle N \rangle (N_{start}) = \langle N \rangle (60) + \int_{60}^{N_{start}} \frac{dV}{dN} (\ N)\, dN \ ;$$

where we set $\langle N \rangle (60) = 1$ and slow roll applies to the 2nd term.

B_{HM} constant at early times, so that

$$\langle N \rangle (N_{start}) \quad 1 + \frac{4e^{\ B_{HM}}}{3} N_{start} \ :$$

Contributing if N_{start} & $e^{B_{HM}}$ 10^{60} 60 e-folds but not in nite.

Conclusions

- Consistent inclusion of 1-loop curvature corrections beyond dS $\rightarrow$ most accurate bounds yet:

$$_{\text{EW}} \gtrsim 0.06$$

independent of $V(\;)$ and N_{start}, m_t-dependent, with bubbles forming close to the end of inflation and hinting against eternal inflation.

- Next step: Starobinsky Inflation (R^2-model):

$$S = \int d^4x \sqrt{\;g_J} \; \frac{M_P^2}{2} \; 1 \quad \frac{h^2}{M_P^2} \; R_J + \frac{1}{12\;^2} R_J^2 + \frac{1}{2} g_J \; \partial \; h \partial \; h \quad \frac{}{4} h^4$$

Starobinsky/R^2 inflation

$$L \quad \frac{M_P^2}{2}R + \frac{1}{2}@ \quad @ \quad + \frac{1}{2}@ \quad @ \qquad V^{\text{total}}(\ ; \ ; R)$$

with $V^{\text{total}} = V_{\text{Star}} + m_{\text{e}} \ \frac{2}{2} + \ {}_{\text{e}} \ \frac{4}{4}$, where $= \quad \frac{1}{6}$ and

$$V_{\text{Star}} = \frac{3 \ {}^2 M_P^4}{4} \quad 1 \quad e^{\ \sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \ {}^2 ;$$

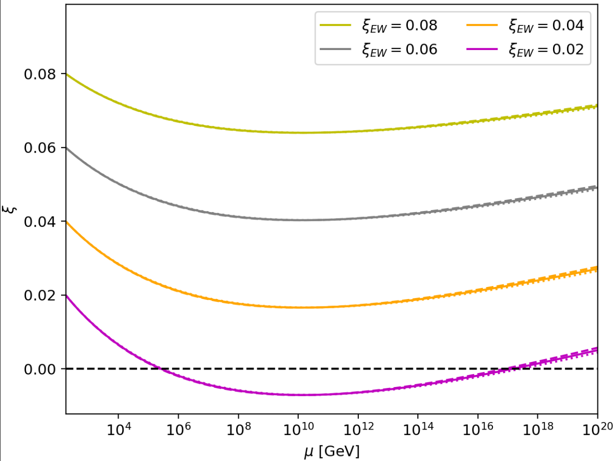
$$m_{\text{e}} = R + 3 \ {}^2 M_P^2 \quad 1 \quad e^{\ \sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \quad e^{\ \sqrt{\frac{2}{3}}\frac{\phi}{M_P}} + \frac{}{M_P^2}@ \quad @ \quad ;$$

$${}_{\text{e}} = \quad + 3 \ {}^2 \ {}^2 e^{\ 2 \ \sqrt{\frac{2}{3}}\frac{\phi}{M_P}} + \frac{4 (\ R)_{\text{e}}}{{}^2 M_P^2} + \frac{4 \ {}^3}{M_P^4}@ \quad @ \quad :$$

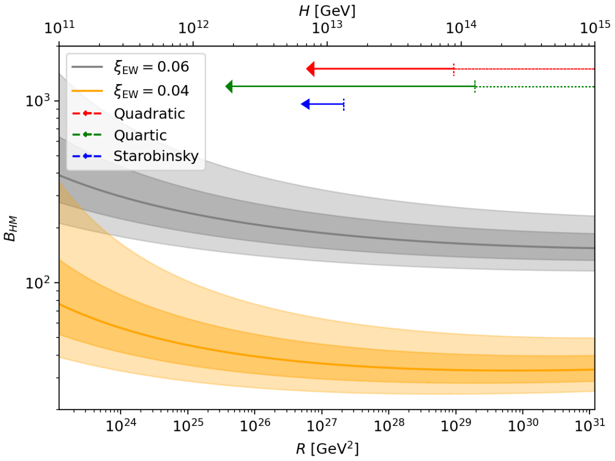
Additional slides

Additional slides - Running of non-minimal coupling

$$16\ {}^2 \quad = 16\ {}^2 \frac{d}{d\ln} = \qquad \frac{1}{6} \quad 12 \quad + 6y_t^2 \quad \frac{3}{2}g'^2 \quad \frac{9}{2}g^2$$



Additional slides - Curvature effects on the bounce action



Shaded areas: 1σ, 2σ deviation from the central m_t; a heavier top quark decreases the value of $B_{\rm HM}$ and vice versa.

Solid red, blue and green arrows: last 60 e-foldings in quadratic, Starobinsky and quartic inflation.