PBHs as DM: formation mechanism and a summary of the existing constraints

Mauro Pieroni

Imperial College London

mpieroni@ic.ac.uk

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Overview

PBH formation: the general picture

If large scalar fluctuations (which requires large Δ_s^2) in the radiation dominated Universe may trigger a collapse leading to the formation of PBHs.

S. W. Hawking 1971

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Mass fraction of the Universe collapsing in PBHs of mass M_{PBH} at time t:

$$\beta_t(M_{PBH}) = \int_{\delta_c}^{1} \frac{M_{PBH}(\delta, t)}{M_H(t)} P_t(\delta) d\delta \ .$$

B. J. Carr 1975; B. J. Carr, J. H. Gilbert, and J. E. Lidsey 1994 (astro-ph/9405027);
A. M. Green and A. R. Liddle 1997 (astro-ph/9704251)

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Requires knowledge on:

- The horizon mass at time t
- The mass which collapses as a function of δ at time t
- The threshold for the collapse δ_c
- The probability distribution of the density contrast δ

PBH formation I

The mass of the horizon at the time t is:

$$M_H(t) = \frac{4\pi}{3}\rho H^{-3} \simeq \left(g_{*,eff}(t)/g^0_{*,eff}\right)^{1/6} (a(t)/a_0)^2 \, M_H^0 \, ,$$

(0 denotinq quantity evaluated today) where we have used:

- $\rho = 3H^2 M_{Pl}^2$ to re-express H
- $\sum_A g^A_{*,s} \equiv g_{*,s} \simeq g_{*,eff} \equiv \sum_A g^A_{*,eff}$
- Entropy conservation: implying $T \propto a^{-1} g_{*,eff}^{-1/3}$ and $\rho \propto g_{*,eff}^{-1/3} a^{-4}$

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In general the mass of the PBHs at formation can be expressed as:

$$M_{PBH}(t,\delta) \simeq \gamma(\delta,t)\, M_H(t)\ ,$$

where for different $\gamma(\delta,t)$ (and assuming it does not depend on t!) we have

- Monochromatic mass spectrum: $\gamma \sim$ constant B. J. Carr 1975
- Critical collapse mass spectrum: $\gamma \sim a(\delta-\delta_c)^b$ M. W. Choptuik 1992, J. C. Niemeyer and K. Jedamzik 1999 (astro-ph/9901292); C. Gundlach 1999 (gr-qc/0001046)

PBH formation II

Statistics of the fluctuations ultimately determines the collapse probability:

- Gaussian distribution $\Longrightarrow P_t(\delta)d\delta = \frac{1}{\sqrt{2\pi\sigma^2(t)}} e^{-\frac{\delta^2}{2\sigma^2(t)}} d\delta$
- Non-Gaussian distribution $\Longrightarrow$???? (some cases can still be solved)

The variance of fluctuations (smoothed on scales $R = (aH)^{-1}$) $\Longrightarrow$ $\sigma^2(R(t)) = \int_0^\infty \frac{\mathrm{d}k}{k}\, \tilde{W}(kR)\Delta_\delta^2(k)$,

where typically $\tilde{W}(kR) = \exp(-k^2R^2/2)$ is used for the smoothing.

Different window functions may affect the result!

K. Ando, K. Inomata and M. Kawasaki 2018 (1802.06393)

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K. Ando, K. Inomata and M. Kawasaki 2018 (1802.06393)

Accurate determination of δ_c is required $\Longrightarrow$ Numerical simulations $\delta_c \simeq 0.4$

D.K. Nadezhin, I.D. Novikov and A. G. Polnarev 1978, I. Musco, J. C. Miller, and L. Rezzolla, 2005, (gr-qc/0412063); T. Harada, C. Yoo, K. Kohri 2013 (1309.4201)

Two comments on the impact of the curvature perturbation profile:

- x-space vs. k-space $\Longrightarrow$ Peak theory computations

  A. M. Green, A. R. Liddle, K. A. Malik and M. Sasaki 2004 (astro-ph/0403181); S. Young, C. T. Byrnes and M. Sasaki, 2014 (1405.7023); C. Germani and I. Musco 2018 (1805.04087)
- Non sphericity may affect the precise value of the threshold

  M. Y. Khlopov, A. G. Polnarev 1980 , A. G. Polnarev and I. Musco 2007 (gr-qc/0605122); F. Kühnel and M. Sandstad 2016 (1602.048)

From formation to present time

The mass fraction of the Universe in PBHs at formation is:

$$\beta_t(M_{PBH}) = \frac{M_{PBH} n_{PBH(t)}}{\rho_{tot}(t)} \ .$$

B.J. Carr, K. Kohri, Y. Sendouda, J. Yokoyama 2009 (0912.5297)

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Assuming adiabatic cosmic expansion, $n_{PBH}(t)/s(t)$ is conserved.

I can thus get (assuming no accretion nor decay):

$$\Omega_{0,PBH}(M) = \left(\frac{a(t)}{a_0}\right)^{-1} \left(\frac{g_{*,eff}(t)}{g^0_{*,eff}}\right)^{-1/3} \beta_t(M_{PBH}) \ .$$

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PBHs are viable candidates for CDM!

G. F. Chapline 1975; P. Ivanov, P. Naselsky and I. Novikov 1994;
J. García-Bellido, A. D. Linde and D. Wands 1996 (astro-ph/9605094);
B. J. Carr, F. Kuhnel and M. Sandstad 2016 (1607.06077)

$$f(M) \equiv \frac{\Omega_{0,PBH}(M)}{\Omega_{DM,0}} \simeq \frac{\beta(M)}{4\times 10^9} \left(\frac{\gamma}{0.2}\right)^{1/2} \left(\frac{g_*(t)}{10.75}\right)^{-1/4} \left(\frac{M}{M_\odot}\right)^{-1/2} .$$

Constraints on PBHs as DM

Present time abundance must be compared with observational constraints!

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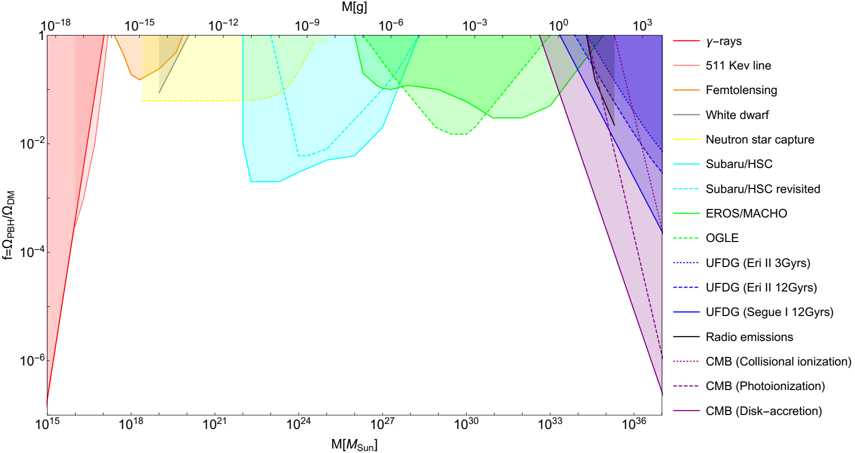
PBHs with $M \lesssim 10^{15} g$ evaporate through Hawking radiation



May change for different mass spectra!

B. Carr, M. Raidal, T. Tenkanen, V. Vaskonen, and H. Veermäe 2017 (1705.05567)

γ-rays constraints

Small PBHs emit via Hawking radiation.

Compare with isotropic (extragalactic) γ-ray background (0.1MeV-10GeV):

- EGRET 2004 (astro-ph/0405441)
- Fermi-LAT 2010 (1002.3603)
- Comptel 2000 (astro-ph/0012332)

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B. J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama 2009 (0912.5297)

Bounds are slightly stronger at low masses (galactic γ-rays)

B. J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama 2016 (1604.05349)

Bounds can be roughly 1-2 orders of magnitude stronger if spin is included

A. Arbey, J. Auffinger and J. Silk 2019 (1906.04750)

Bounds can be roughly 1 order of magnitude stronger if we assume some knowledge of the source (AGN and blazers)

G. Ballesteros, J. Coronado-Blázquez, D. Gaggero 2019 (1906.04750)

511KeV line constraints

PBHs emit high energy radiation among which positrons.

Positrons annihilate with electrons

Decaying product $\Longrightarrow$ two γ-rays with energy of 511 keV each.

Compare with direct measurement:
Spectrometer SPI on ESA's INTEGRAL observatory (1512.00325)

W. DeRocco, P. W. Graham 2019 (1906.07740)

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W. DeRocco, P. W. Graham 2019 (1906.07740)

Requires assumptions on positron propagation in the Galaxy (in the ISM):

Positrons do not propagate much farther than 1 kpc from their source.

Justified for $E \lesssim$ 1MeV, stronger bound if extrapolated to $E \gtrsim$ 1MeV.

Femtolensing constraints

The spectrum of γ-rays traveling close to PBHs is distorted (by lensing).

Compare with γ-ray burst observations (8KeV - 40MeV):
FERMI Gamma-Ray Burst Monitor 2011 (1104.5495)

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Assumes the source to be point-like (meaning $L \sim 10^6$m) in the lens plain!

The typical sizes can actually be quite larger:

- $L \sim 10^8$m -10^9m A. Barnacka, A. Loeb 2014 (1409.1232)
- $L \sim 10^{11}$m -10^{12}m V. Z. Golkhou, N. R. Butler, O. M. Littlejohns. 2015 (1501.05948)

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Bounds are weaker ($\sim$ one order of magnitude)
A. Katz, J. Kopp, S. Sibiryakov 2018 (1807.11495)

More observations are necessary to be at the same level
A. Katz, J. Kopp, S. Sibiryakov 2018 (1807.11495)

No bounds with present data
P. Montero-Camacho, X. Fang, G. Vasquez, M. Silva, C. M. Hirata 2019 (1906.05950)

White dwarf constraints

PBHs passing close to (or inside) a White Dwarf gives it energy by friction.

This energy injection may trigger fusion.

WD with $M \lesssim 1.25 M_\odot$ are eventually destroyed by this process (supernovae).

Compare with observed abundance of WD:

Sloan Digital Sky Survey 2012 (1212.1222) makes a WD catalogue.

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Requires an estimate of the collision rate.

Recent works claim it is overestimated $\Longrightarrow$ No bounds

P. Montero-Camacho, X. Fang, G. Vasquez, M. Silva, C. M. Hirata 2019 (1906.05950)

Neutron star capture constraints

Neutron star captures a passing PBH and eventually gets eaten.

Compare with observed abundance of stars in Globular Clusters:

ACS on board the Hubble Space Telescope (HST) 2009
(0804.2025, 0911.2469)

F. Capela, M. Pshirkov, P. Tinyakov 2013 (1301.4984)

Analogous arguments hold for PBH capture during star formation.

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Assumes NS to be located in GC and estimates capture probability

Recent works claim it is overestimated $\Longrightarrow$ No bounds

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Subaru/HSC constraints

Microlensing when PBHs crosses (or close to) the line of sight to some star.

Compare with observed microlensing events from stars:

Subaru Hyper Suprime-Cam monitors Andromeda galaxy(M31).

H. Niikura *et al.* 2017 (1701.02151)

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H. Niikura *et al.* 2017 (1701.02151)

Assumes knowledge on the characteristic size of the stars

Recent work claim it is underestimated!

Constraints $\sim$ three orders of magnitude weaker on small masses

N. Smyth, S. Profumo, S. English, T. Jeltema, K. McKinnon and P. Guhathakurta 2019 (1910.01285)

EROS/OGLE/MACHO constraints

Microlensing when PBHs crosses(or close to) the line of sight to some star.

Compare with observed microlensing events from stars:

- EROS 1 (supernovae search at z$\sim 0.02 - 0.2$) 2000 (astro-ph/0006424)
- MACHO (Large Magellanic cloud) 2001 (astro-ph/0011506)
- EROS 1 (Small Magellanic cloud) 2002 (astro-ph/0212176)
- EROS 2 (Small and Large Magellanic cloud) 2006 (astro-ph/0607207)
- OGLE (inner Galactic Bulge and Magellanic System)
  A. Udalski *et al.* 1994 (1994ApJ, 426)
- OGLE (inner Galactic Bulge and Magellanic System)
  A. Udalski, M. K. Szymański, G. Szymański 2015 (1504.05966)

Microlensing mainly explained by stars $\Longrightarrow$ constraints!

EROS-2 2006 (astro-ph/0607207), H. Niikura, M. Takada, S. Yokoyama, T. Sumi, S. Masaki 2019 (1901.07120)

Ultra faint dwarf galaxies constraints

PBHs passing close to (or through) a UFDG (small-low-luminosity galaxy)

The stars gain energy and eventually escape the galaxy attraction

Ultimately UFDG are destroyed if too much massive BHs are around

Compare with observed abundance of star cluster in UFDGs:

- Keck/DEIMOS (Segue 1) M. Geha *et al.* 2008 (0809.2781)
- Keck/DEIMOS (Segue 1) J. D. Simon *et al.* 2010 (1007.4198)
- DES Collaboration 2015 (1503.02584)
- DES data, S. E. Koposov *et al.* 2015 (1503.02079)
- Panoramic Survey Telescope and Rapid Response System 1 (Pan-STARRS1) B. P.M. Laevens *et al.* 2015 (1507.07564)

T. D. Brandt 2016 (1605.03665), S. M. Koushiappas and A. Loeb 2017 (1704.01668)

The constraints depend on the assumption on the age of the UFDG!

Radio emission constraints

Interstellar gas may eventually form an accretion disk for PBHs

While accreting the PBH will emit a broad spectrum of radiation

Compare with radio catalogs:
VLA radio catalog at 1.4 GHz J. W. Lazio and J. M. Cordes 2008
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Same could be done with X-rays (but bound is weaker):

- Chandra catalog (0.5keV - 8keV) M. P. Muno *et al.* 2009 (0809.1105)
- NuSTAR (10keV - 40keV) F. A. Harrison *et al.* 2013 (1301.7307)

D. Gaggero, G. Bertone, F. Calore, R. M. T. Connors, M. Lovell, S. Markoff and E. Storm 2016 (1612.00457)

CMB constraints

While PBH is accreting, part of the energy is emitted as radiation

Injection of energy in the primordial plasma

affects thermalization and ionization history

Look for signatures in the CMB:

- WMAP M. Ricotti, J. P. Ostriker, K. J. Mack 2007 (0709.0524)
- Planck L. Chen, Q. G. Huang, K. Wang 2016 (1608.02174)
- Planck (update with new astro modeling)
  Y. Ali-Haïmoud, M. Kamionkowski 2016 (1612.05644)
- Alternative approach but again Planck data
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All these constraints assume spherical accretion (simpler and more robust)

Bounds are stronger assuming accretion disk!

V. Poulin, P. D. Serpico, F. Calore, S. Clesse, K. Kohri 2017 (1707.04206)

Inflation: the basic picture

Einstein Equations for an homogeneous and isotropic background ($\Lambda = 0$):

$$3\left(\frac{\dot{a}}{a}\right)^2 \equiv 3H^2 = \kappa^2\rho, \qquad -2\dot{H} = \kappa^2(p+\rho) ,$$

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To solve problems with early times :
we introduce inflation as an early phase of nearly exponential expansion.

A. Guth 1981, A. D. Linde 1982, A. Albrecht and P. J. Steinhardt 1982

An exponentially increasing scale factor $a \propto \exp(Ht)$ is obtained for:
$p = -\rho = const$ (Like CC)
This corresponds to static solution (*i.e.* dS spacetime)!

Not our Universe! Inflation must end!
Dynamical implementation $\longrightarrow$ departure from $p \simeq -\rho \simeq const$!

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At the end of inflation the Universe is cold and empty.
(Because of the expansion ρ of the other species goes to zero!)
We need a connection with the other phases of evolution!
Reheating $\longrightarrow$ Explosive decay of the inflaton to repopulate the Universe.

Slow-roll inflation

Homogeneous scalar field ϕ in a homogeneous and isotropic universe :

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{\dot{\phi}^2}{2} - V(\phi)\right).$$

In order to realize inflation we need:

$$p \simeq -\rho \simeq const \qquad \Longleftrightarrow \qquad \left|\frac{\dot{\phi}^2}{2}\right| \ll |V| \sim const$$

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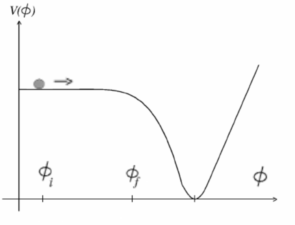


$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2} \simeq \frac{1}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \equiv \epsilon_V ,$$

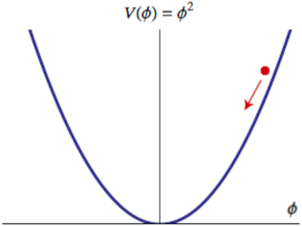
$$\epsilon_2 \equiv \frac{\mathrm{d} \ln(\epsilon_1)}{\mathrm{d} \ln a} \simeq -2\eta_V + 4\epsilon_V ,$$

$$N(t) \equiv \int_{a_f}^{a} \mathrm{d} \ln \hat{a} \simeq -\int_{\phi_f}^{\phi} \frac{\mathrm{d}\hat{\phi}}{\sqrt{2\epsilon_V}}$$

An explicit example: Chaotic inflation

Let us consider the case of Chaotic inflation: $V(\phi) = m^2\phi^2/2$

A. D. Linde. Chaotic Inflation 1983



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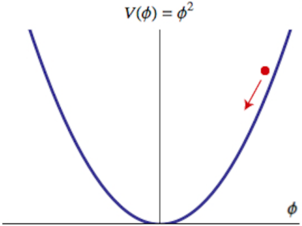

The slow roll parameters read:

$$\epsilon_V = \frac{2}{\phi^2}\,, \qquad \eta_V = \frac{2}{\phi^2}\,,$$

and the number of e-folding is:

$$N \simeq -\int_{\phi_f}^{\phi} \frac{V(\hat{\phi})}{V_{,\phi}(\hat{\phi})} \mathrm{d}\hat{\phi} \simeq \frac{\phi^2}{4}\,.$$

Cosmological perturbations

We can consider perturbations around the homogeneous background:

$$\Phi(t,\vec{x}) = \phi(t) + \delta\phi(t,\vec{x}) \ , \qquad \mathbf{g}_{\mu\nu}(t,\vec{x}) = g_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\vec{x}) \ ,$$

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After painful computations the eom for gauge invariant quantities are:

$$\tilde{v}''_{\vec{k}} + \left(k^2 - \frac{z''}{z}\right)\tilde{v}_{\vec{k}} = 0 \,, \qquad \tilde{v}''_{\alpha,\vec{k}} + \left(k^2 - \frac{a''}{a}\right)\tilde{v}_{\alpha,\vec{k}} = 0 \,,$$

where $'$ denotes a derivative with respect to $\tilde{\tau}$:

$$\tilde{\tau} \equiv c_s \tau \equiv \frac{c_s}{a}\mathrm{d}t \,, \qquad \tau = \int \frac{\mathrm{d}a}{a^2 H} \simeq -\frac{1}{aH} \,.$$

where q is some quantity and we have introduced:

$$z \equiv \frac{a\sqrt{2\epsilon_H}}{c_s} \,, \qquad c_s^2 \equiv \left(\frac{p+\rho}{2X\rho_{,X}}\right) \,, \qquad X \equiv \frac{g^{\mu\nu}}{2}\partial_\mu\phi\partial_\nu\phi \,.$$

Cosmological perturbations

We can consider perturbations around the homogeneous background:

$$\Phi(t,\vec{x}) = \phi(t) + \delta\phi(t,\vec{x}) \,, \qquad \mathbf{g}_{\mu\nu}(t,\vec{x}) = g_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\vec{x}) \,,$$

After painful computations the eom for gauge invariant quantities are:

$$\tilde{v}_{\vec{k}}'' + \left(k^2 - \frac{z''}{z}\right)\tilde{v}_{\vec{k}} = 0 \,, \qquad \tilde{v}_{\alpha,\vec{k}}'' + \left(k^2 - \frac{a''}{a}\right)\tilde{v}_{\alpha,\vec{k}} = 0 \,,$$

where $'$ denotes a derivative with respect to $\tilde{\tau}$:

$$\tilde{\tau} \equiv c_s \tau \equiv \frac{c_s}{a} \mathrm{d}t \,, \qquad \tau = \int \frac{\mathrm{d}a}{a^2 H} \simeq -\frac{1}{aH} \,.$$

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At the lowest order $z''/z \simeq a''/a \simeq 1/\tilde{\tau}^2$.

- **Short wavelength** ($1 \ll k^2\tilde{\tau}^2$): $\longrightarrow \tilde{v}_{\vec{k}} \propto \exp(\pm i k c_s \tau)$
- **Long wavelength** ($k^2\tilde{\tau}^2 \ll 1$): $\longrightarrow \tilde{v}_{\vec{k}} \simeq C_{1,k} z + C_{2,k} z \int \frac{\mathrm{d}\tilde{\tau}}{z^2} \propto 1/\tau$

CMB Observables

Scalar and tensor (dimensionless) power spectra (2-point functions) are:

$$\Delta_s^2(k,\tau)\Big|_{\tau=k^{-1}} = \frac{1}{8\pi^2}\frac{H^2}{\epsilon_H}\,, \qquad \Delta_t^2(k,\tau)\Big|_{\tau=k^{-1}} = 2\left(\frac{H}{\pi}\right)^2\,.$$

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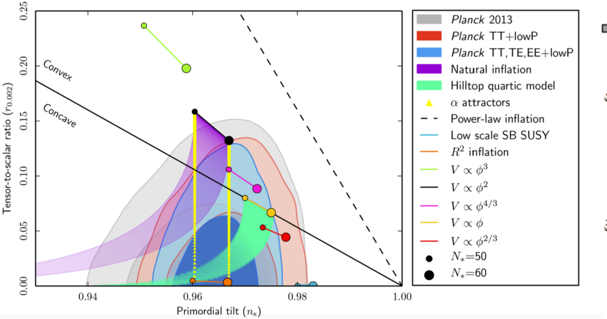
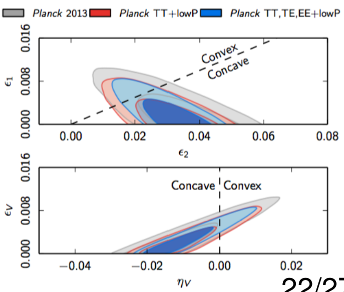
Tensor-to-scalar ratio (r) and the scalar spectral index (n_s):

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2}\Big|_{k=aH} \simeq 16\epsilon_V\,, \qquad n_s \equiv 1 + \frac{\mathrm{d}\ln\Delta_s^2(k)}{\mathrm{d}\ln k}\Big|_{k=aH} \simeq 1 + 2\eta_V - 6\epsilon_V\,.$$





Enhanced power spectrum

The equation of motion for curvature perturbation $\mathcal{R}_k = v_k/z$ is:

$$\mathcal{R}_k'' + 2\frac{z'}{z}\mathcal{R}_k' + k^2\mathcal{R}_k = 0\,,$$

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which in terms of the number of e-folds ($\mathrm{d}N = H\mathrm{d}t = \frac{aH}{c_s}\mathrm{d}\tilde{\tau}$) reads:

$$\frac{\mathrm{d}^2\mathcal{R}_k}{\mathrm{d}N^2} + \left(3 - \epsilon_H + \epsilon_2 - 2s_1\right)\frac{\mathrm{d}\mathcal{R}_k}{\mathrm{d}N} + \frac{c_s^2k^2}{a^2H^2}\mathcal{R}_k = 0.$$

where $\epsilon_H \equiv -H_{,N}/H$, $\epsilon_2 \equiv \epsilon_{H,N}/\epsilon_H$ and $s \equiv c_{s,N}/c_s$,

whose solution for long wavelengths is:

$$\mathcal{R}_k \simeq C_{1,k} + 2\,C_{2,k}\int\frac{\mathrm{d}\tilde{\tau}}{z^2} = C_{1,k} + C_{2,k}\int\frac{c_s^2}{a^3\epsilon_H H}\mathrm{d}N\,.$$

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The condition $c_s k = aH$ sets the horizon crossing

Fast decrease in c_s may push inside of the horizon modes which already left!

They restart oscillating (before crossing again) leaving a particular signature!

G. Ballesteros, J. Beltran Jimenez, M. Pieroni 2018 (1811.03065)

Non-standard kinetic terms

A generalized action for the inflaton can be written as:

$$S = \int \mathrm{d}^4 x \sqrt{-g}\mathcal{L}(X,\phi) \,, \qquad \text{where} \qquad X \equiv \frac{g^{\mu\nu}}{2}\partial_\mu\phi\partial_\nu\phi \,.$$

The energy density and pressure can be expressed as:

$$p(X,\phi) = \mathcal{L}(X,\phi), \qquad \rho(X,\phi) = -2Xp_{,X}(X,\phi) - p(X,\phi) \,.$$

Then it is possible to express $-\dot{H}/H^2$ as:

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Some realizations are:

1. K-inflation: C. Armendariz-Picon, T. Damour, and Viatcheslav F. Mukhanov, 1999
2. DBI Inflation: Eva Silverstein and David Tong 2004
3. Tachyonic inflation: Eva Silverstein and David Tong 2002

May predict large non Gaussianities and enhanced Δ_s^2!

A single-field toy model

Let us consider the DBI-lagrangian:

$$p(\phi, X) = -V + \left(1 - \sqrt{1 - 2f\,X}\right) f^{-1}$$

where the potential and the warp factor are respectively:

$$V(\phi) = V_0 \left(1 - y\frac{M_P^2}{\phi^2}\right)^2 , \qquad \frac{1}{f(\phi)} = q\,M_P^4 + Q\,(\phi - \phi_0)^4 .$$

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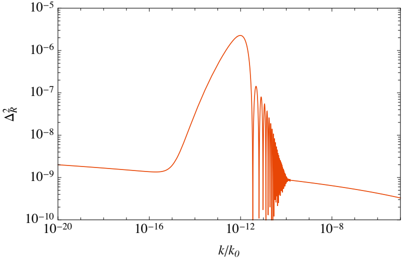

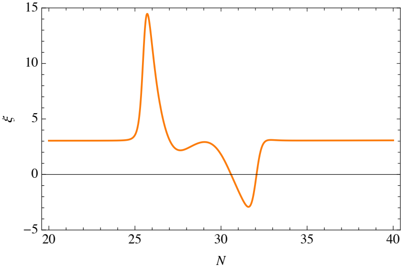

G. Ballesteros, J. Beltran Jimenez, M. Pieroni 2018 (1811.03065)

Conclusions and future perspectives

What we know so far:

- PBHs are very interesting for cosmology
- Formation is affected by many uncertainties
- Bunch of constraints exist (also many uncertainties)
- If inflationary, open a window on scales very far from CMB

How to proceed:

- Critical aspects of the formation mechanism?
- Detailed study of the evolution after formation?
- Reconsider existing constraints?
- Explore new ways to probe them?
- More (inflationary) models?

Last Slide

The End

Thank you

CMB inflationary constraints

The most stringent constraints (Planck 2015) are (all at $N \simeq 60$):

- **COBE Normalization**: Sets the value of the scalar power spectrum at the CMB scales:

$$\Delta_s^2\Big|_{N_{CMB}} = (2.21 \pm 0.07) \cdot 10^{-9} \ .$$

- **Planck constraints on n_s and r**: tilt of the scalar specrtum and amount of GWs produced during inflation:

$$n_s = 0.9645 \pm 0.0049 \ , \qquad\qquad r < 0.10 \ .$$

- **Non gaussianities**: Three point function for cosmological perturbations are small:

$$\langle \zeta(\tau, \vec{k}_1)\zeta(\tau, \vec{k}_2)\zeta(\tau, \vec{k}_3)\rangle = (2\pi)^3 \, B(k_1, k_2, k_3)\delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \ .$$

Typically constraints are set on f_{NL} (for actual values see Planck 2015):

$$B(k_1, k_2, k_3) \equiv f_{NL} F(k_1, k_2, k_3)$$

21cm line constraints

Parallel spin configuration in neutral hydrogen is slightly more energetic that anti-parallel ($\sim$5.87μeV) $\Longrightarrow$ very low transition rate (2.9×10^{-15}s^{-1})

Characteristic radiation at $f \sim$ 1.4GHz (corresponding to $\lambda \sim$ 21cm)

to be observed (after redshift) at $f \sim$ 200MHz$-$9MHz

Observation would be relevant to:

- Determine Ω_m (mainly through the intensity)
- Map structures (reionization of hydrogen $\Longrightarrow$ holes in the background)

Constraints from future experiments (in range $1 - 10^3 M_\odot$):

- Hydrogen Epoch of Reionization Array (HERA) D. R. DeBoer *et al.* 2016 (1606.07473)
- Square Kilometre Array (SKA) G. Mellema *et al.* 2012 (1210.0197)

O. Mena, S. Palomares-Ruiz, P. Villanueva-Domingo and S. J. Witte 2019 (1906.07735)

Also constraints on smaller masses (same as γ-ray bounds)

K. J. Mack and D. H. Wesley 2008 (0805.1531), B. J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama 2009 (0912.5297)

Some numbers

- The Sun mass is $M_\odot \simeq 2 \times 10^{33}$g
- The (reduced) Planck mass is $m_{PL} \simeq 4.3 \times 10^{-6}$g $\simeq 2.2 \times 10^{-39} M_\odot$
- The Hubble parameter at present time is $H_0 = 100\ h \frac{\text{km}}{\text{Mpcs}} \simeq 3.24\ h \times 10^{-18}$1/s
- The Planck time is $t_{PL} \simeq 5.39 \times 10^{-44}$
- We work in natural units implying $t_{PL} * m_{PL} = \frac{1}{\sqrt{8\pi}} \simeq \frac{1}{5}$
- The Hubble parameter at present time is $H_0 \simeq 1.7\ h \times 10^{-61} 1/t_{PL}$
- The horizon mass is $M_H = \frac{4\pi}{3} \rho H^{-3} = 4\pi m_{PL}^2 H^{-1}$
- The horizon mass today is:

$$M_H \simeq 4.2 \times 10^{22} \left(\frac{0.7}{h} \right) M_\odot\ .$$

- CMB scales are $k_* \simeq 0.05 Mpc^{-1}$
- The mass of a PBH formed when scale k collapses is roughly:

$$M_{PBH} \simeq 10^{-15} M_\odot \left(\frac{k}{10^{14}\ \text{Mpc}^{-1}} \right)^{-2} .$$